

# Combinatorial approaches to directed spaces for rewriting and concurrency

# Summary

Different notions of "directed space" have arisen independently in different areas of theoretical computer science such as concurrency and rewriting theory, suggesting a new topological perspective on aspects of computation. This project will try to connect and refine some of the earlier approaches to directed topology through a new combinatorial framework, the theory of diagrammatic sets. The goal is to port and extend invariants defined in concurrency theory to rewriting, and vice versa; study their computability; understand the role of "higher-dimensional direction" in concurrent systems; and learn something about the abstract structure of "directed homotopy theory".

Research field:	Information and communication technology
Supervisors:	Prof. Dr. Pawel Maria Sobocinski
	Prof. Dr. Amar Hadzihasanovic
Availability:	This position is available.
Offered by:	School of Information Technologies
	Department of Software Science
Application deadline:	Applications are accepted between October 01, 2022 00:00 and October 23, 2022 23:59 (Europe/Zurich)

# Description

In recent decades, new approaches have arisen in different areas of theoretical computer science where computational structures are viewed under a topological lens as "directed spaces", that is, spaces where paths or "higher-dimensional paths" have a preferred orientation and are non-reversible. These approaches can give access to powerful methods of modern homotopy theory and higher category theory in studying computational properties via functorial, that is compositional invariants.

In particular, an approach to concurrency theory has focussed mainly on "point-set" models of spaces with directed paths, such as pospaces and dispaces; and, in this context, studied and defined interesting invariants, such as natural homotopy and homology.

By contrast, the polygraph approach to rewriting theory has identified various types of rewrite systems with instances of an algebraic model of "directed cell complexes", which not only have directed paths (1-cells) but also directed higher paths (n-cells). This has led to new quantitative results in computational universal algebra based on homology theory.

Recently, a combinatorial framework for directed spaces in rewriting theory – the theory of diagrammatic sets – has been proposed, which combines the expressiveness of the polygraph approach with better computational properties and stronger ties to standard topological models.

The goal of this PhD project is to study the application of this model to concurrency theory and its relation to other models used therein; and, more in general, to further the "directed space" perspective in concurrency and rewriting, both separately and as a bridge between the two.

Some of the questions that the thesis may address are:

1) Can we define interesting functors (or better adjunctions) between diagrammatic sets and other models of directed spaces?

2) Can we import invariants defined for point-set or other models into the combinatorial framework?

3) Can we extend these invariants to ones that are sensitive to the direction of higher-dimensional cells? What information do these give about rewrite systems?

4) How can we compute these invariants algorithmically in the combinatorial framework?

5) What role do higher-dimensional directions play in the analysis of concurrent systems?

6) Can we learn something from this comparison about the fundamental structures of "abstract directed homotopy theory"?



According to the candidate's inclination, the project may also, or alternatively, explore aspects related to higher categories, homotopy theory, or models of computation. A part of the work may be formalised or implemented in proof assistants.

The PhD candidate will join the Compositional Systems and Methods group at Tallinn University of Technology, and be supervised by Dr Amar Hadzihasanovic (main supervisor) together with Prof Pawel Sobocinski (co-supervisor).

## **Responsibilities and (foreseen) tasks**

- · Study and compile a thorough literature review on the relevant research areas
- Produce original research and disseminate it through journal and conference papers and presentations
- Participate in group activities of the Compositional Systems and Methods group, including weekly meetings, research seminars, and retreats
- Do relevant coursework, training, and service activities, as required of a PhD candidate

#### Applicants should fulfil the following requirements:

- a master's degree in mathematics, computer science, or a closely related field
- a grounding in one or more of category theory, homotopy theory, concurrency theory, or rewriting theory
- excellent command of English
- · strong and demonstrable writing and analytical skills
- · capacity and willingness to provide assistance in organizational tasks relevant to the project

#### The following experience is beneficial:

- previous experience with directed algebraic topology, higher category theory, or higher-dimensional rewriting theory
- some programming experience

The candidate should submit a motivation letter detailing their research interests and experience, and how they align with the proposed project.

We encourage applications from members of under-represented group, and will provide support to applicants who are disadvantaged due to factors including, but not limited to gender, ethnicity, sexuality, disability, and socio-economic status.

#### We offer:

- 4-year fully funded PhD position in a dynamic, cross-cutting group working at the interface of mathematics and computer science
- Opportunities for funded conference visits, summer schools, research stays and networking with globally leading universities and research centres

#### About the department

The Laboratory for Compositional Systems and Methods was established in 2019 at TalTech to support interdisciplinary research at the interface of mathematics and computer science. Led by Prof Pawel Sobocinski and funded with a  $\leq$ 1.6 million grant from the research initiative of the Estonian IT Academy, it is emerging as a leading hub for a community of researchers using compositional, categorical and diagrammatic methods.

Beside Prof Sobocinski, the group hosts 3 postdoctoral researchers, including Dr Amar Hadzihasanovic, and 6 doctoral students. The group's ethos emphasises openness, and inter-group collaboration is highly encouraged. Members participate in regular group meetings and occasional research retreats in the Estonian countryside. It also works closely alongside the Logic and Semantics group, with which it shares a weekly research seminar.



The group is well-supported by the School of Information Technologies at TalTech, which will provide the researcher with the necessary infrastructure. The Research Administration Office supports all researchers in running their projects, applying for research funding, and finding partners. The International Staff Centre, under the Human Resources Department, is there to provide the necessary assistance to foreign workers before and after their arrival in Estonia, helping them enter and socialise in the university and in Estonia in general. Moreover, the International Staff Centre coordinates in-house training which is held in English (didactics and pedagogy workshops and courses, cultural training, social and communicative skills training, language courses for Estonian at level A1-B1) and organises an Introduction Day for International Staff. TalTech has a compact campus with modern and sophisticated technological infrastructure. TalTech's library provides access to a large range of academic databases and to several research databases and repositories.

## Additional information

For further information, please contact Dr Amar Hadzihasanovic (amar@ioc.ee) or Prof Pawel Sobocinski, and visit https://compose.ioc.ee/



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